CHAPTER 21
SYNTHETIC APERTURE RADAR

L. J. Cutrona
Sarcutron, Inc.

21.1 BASIC PRINCIPLES AND EARLY HISTORY

For airborne ground-mapping radar there has been continuous pressure and desire to achieve finer resolution. Initially, this finer resolution was achieved by the application of "brute-force" techniques. Conventional radar systems of this type were designed to achieve range resolution by the radiation of a short pulse and azimuth resolution by the radiation of a narrow beam.

The range resolution problem and some of the pulse compression techniques are discussed in Chap. 10. There it is shown that techniques are available for achieving a resolution significantly finer than that corresponding to the pulse width, provided a signal of sufficient bandwidth is transmitted. Since pulse compression is adequately treated in that chapter, the present chapter will discuss pulse compression techniques only for cases in which the pulse compression technique is intimately involved with synthetic aperture techniques. This is particularly true for configurations that perform both pulse compression and azimuth compression simultaneously rather than with techniques that perform range compression and azimuth compression sequentially.

The basic technology discussed in this chapter is the exploitation of synthetic aperture techniques for improving the azimuth resolution of a mapping radar to a value significantly finer than that achievable by making use of the radiated beamwidth.

Synthetic aperture radar (SAR) is based on the generation of an effective long antenna by signal-processing means rather than by the actual use of a long physical antenna. In fact, only a single, relatively small, physical antenna is used in most cases.

In considering a synthetic aperture, one makes reference to the characteristics of a long linear array of physical antennas. In that case, a number of radiating elements are constructed and placed at appropriate points along a straight line. In the use of such a physical linear array, signals are fed simultaneously to each of the elements of the array. Similarly, when the array is used as a receiver, the elements receive signals simultaneously; in both the transmitting and the receiving modes, waveguide or other transmission-line interconnections are used, and interference phenomena are exploited to get an effective radiation pattern.

The radiation pattern of a linear array is the product of two quantities if the radiating elements are identical. The radiation pattern of the array is the radiation pattern of a single element multiplied by an array factor. The array factor has significantly sharper lobes (narrower beamwidths) than the radiation patterns of the elements of the array. The half-power beamwidth β, in radians, of the array factor of such an antenna is given by

$$\beta = \frac{\lambda}{L} \tag{21.1}$$

In this expression, L is the length of the physical array, and λ is the wavelength.

In the synthetic antenna* case, only a single radiating element is used in most instances. This antenna is translated to take up sequential positions along a line. At each of these positions a signal is transmitted, and the radar signals received in response to that transmission are placed in storage. It is essential that the storage be such that both amplitude and phase of received signals are preserved.

After the radiating element has traversed a distance L_{eff}, the signals in storage resemble strongly the signals that would have been received by the elements of an actual linear array. Consequently, if the signals in storage are subjected to the same operations as those used in forming a physical linear array, one can get the effect of a long antenna aperture. This idea has resulted in the use of the term *synthetic aperture* to designate this technique.

In the case of an airborne ground-mapping radar system, the antenna usually is mounted to be side-looking, and the motion of the aircraft carries the radiating element to each of the positions of the array. These array positions are the location of the physical antenna at the times of transmission and reception of the radar signals.

The designer of a synthetic aperture radar has available a number of degrees of freedom that are not available to the designer of a physical linear array. These degrees of freedom derive from the fact that the signals in storage can be selected by range and that, if desired, a different operation can be performed on the signals at different ranges. One important operation of this type is that of *focusing*.

A physical linear array can be focused to a specific range. There will then be a depth of focus surrounding this range. However, most physical linear arrays are unfocused. This is sometimes stated by saying that the antenna is "focused at infinity." In a synthetic aperture radar, however, it is possible to focus each range separately by the proper adjustment of the phases of the received signals before the summation; this results in the effective synthetic aperture. Furthermore, if desired, a different weighting can be applied to each range, although usually the same type of weighting is used at all ranges.

There is another important difference between physical linear arrays and synthetic linear arrays. This difference results in the synthetic aperture having a resolution finer by a factor of 2 than that corresponding to a real linear array of the same length. Qualitatively, the following discussion indicates the physics resulting in this factor of 2. In a more general analysis, the factor 2 arises naturally.

In a physical linear array, the transmission of the signals results in an illumination of the target area. The angle selectivity of the linear array is provided only during the reception process. During this process, the differences in phase received by each element of the linear array give the antenna pattern. In the synthetic antenna radar, on the other hand, a single element radiates and receives signals. Consequently, the round-trip phase shift is effective in forming the effective radiation pattern. This relationship is written as

*The terms *synthetic antenna* and *synthetic aperture* are used interchangeably in this chapter.

$$\beta_{\mathrm{eff}} = \frac{\lambda}{2\mathrm{L}_{\mathrm{eff}}} \tag{21.2}$$

Here β_{eff} is the effective half-power beamwidth of the synthetic aperture, and L_{eff} is the length of the synthetic aperture.

A more detailed derivation of the resolution capability of a synthetic aperture radar will be given later in this chapter. The following derivation is that initially made by the author and his colleagues in the early days of synthetic aperture radar.

Let D represent the horizontal aperture of the physical antenna carried by an airborne ground-mapping radar. The width of the horizontal beam at range R gives the maximum value for the length of synthetic aperture that can be used at that range. Since the beamwidth of such an antenna is given by the ratio of the wavelength λ to its horizontal aperture D, the maximum length of this synthetic antenna aperture is given by

$$L_{\mathrm{eff}} = \frac{R\lambda}{D} \tag{21.3}$$

The linear resolution in azimuth δ_α is the product of the effective beamwidth given by Eq. (21.2) and the range R:

$$\delta_\alpha = \beta_{\mathrm{eff}} R \tag{21.4}$$

If Eqs. (21.2) and (21.3) are combined with Eq. (21.4), one obtains

$$\delta_\alpha = \frac{\lambda}{2L_{\mathrm{eff}}} R = \frac{\lambda R}{2} \frac{D}{R\lambda} = \frac{D}{2} \tag{21.5}$$

It will be noted that Eq. (21.5) indicates an azimuth linear resolution independent of both range and wavelength. Moreover, the result indicates that finer resolution is achievable with smaller rather than larger physical apertures. This spectacular result formed much of the motivation of the research in synthetic antenna radar.

The author was first exposed to the idea of a synthetic antenna radar in 1953, during a summer study which launched a program known as Project Michigan. During that summer, the ideas relating to synthetic antennas were presented by Dr. C. W. Sherwin,[1] then of the University of Illinois, Dr. Walt Hausz of the General Electric Company, and J. Koehler, at that time with Philco Corporation. Subsequently, it came to the author's attention that Carl Wiley and the Goodyear Aircraft Company had already undertaken some work and had made substantial progress in the synthetic-antenna area.

The Pioneer Award of the IEEE Aerospace and Electronic Systems Society was given to Carl Wiley in 1985 for his work in synthetic aperture radar. His remarks from that presentation are given in Ref. 2 and relate some of the early history of SAR.

Most of the early workers considered an unfocused synthetic antenna. However, Dr. Sherwin indicated that finer resolution should be achievable by using focusing because this technique removed what would otherwise be a restriction on the maximum length of synthetic antenna that could be used. The author and his colleagues at the University of Michigan undertook development of the focusing concept suggested by Dr. Sherwin.

21.2 FACTORS AFFECTING RESOLUTION OF A RADAR SYSTEM

In the following paragraphs a brief comparison of the conventional antenna, the unfocused synthetic antenna, and the focused antenna is given.[3,4] The language of synthetic apertures is used, and a comparison of the resolution capability for three cases is given. A more sophisticated derivation of simultaneous resolution in range and azimuth will be given later in this chapter.

Three cases are compared for their azimuth resolution capability: (1) the *conventional technique*, in which azimuth resolution depends upon the width of the radiated beam; (2) the *unfocused synthetic antenna technique*, in which the synthetic antenna length is made as long as the unfocused technique permits; and (3) the *focused synthetic antenna technique*, in which the synthetic antenna length is made equal to the linear width of the radiated beam at each range.

The linear azimuth resolution for the *conventional case* is given by

$$\text{Resolution}_{\text{conv}} = \frac{\lambda R}{D} \tag{21.6}$$

For the *unfocused case*, the resolution is

$$\text{Resolution}_{\text{unf}} = \tfrac{1}{2}\sqrt{\lambda R} \tag{21.7}$$

whereas for the *focused case*, the resolution is

$$\text{Resolution}_{\text{foc}} = \frac{D}{2} \tag{21.8}$$

where λ = wavelength of radar signal transmitted
D = horizontal aperture of antenna
R = radar range

Figure 21.1 is a plot of the resolution for each of these cases as a function of radar range. This plot is for an antenna aperture of 5 ft and a wavelength of 0.1 ft.

Conventional Technique. The conventional technique for achieving azimuth resolution has been that of radiating a narrow beam. In this case the *resolution* of a target depends upon whether the target is included within the half-power points of the radiated beam, although some techniques exist for resolving targets somewhat less than a beamwidth apart.

The computation of the linear azimuth resolution for the conventional case is well known. The appropriate expression is obtained by noting that the width of the radiated beam, in radians, is given by the ratio λ/D whereas the linear width of the beam at range R is the product of this beamwidth and range. These considerations lead to the result already written as Eq. (21.6).

A consideration from antenna theory is that Eq. (21.6) applies only to the far-

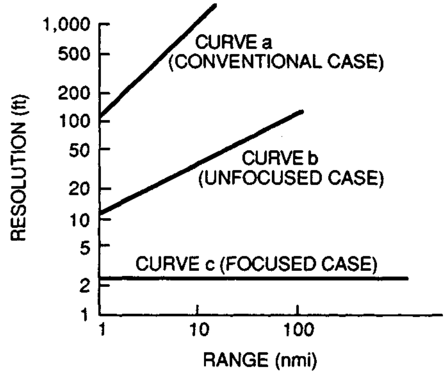

FIG. 21.1 Azimuth resolution for three cases: curve *a*, conventional; curve *b*, unfocused; curve *c*, focused.

field pattern of an antenna. The beginning of the far field occurs at a distance R_{min} for which

$$R_{min} \approx \frac{D^2}{\lambda} \tag{21.9}$$

It will be noted by substitution of Eq. (21.9) that the finest resolution achievable by the conventional technique is given by

$$\text{Minimum conventional resolution} = D \tag{21.10}$$

The Unfocused Synthetic Aperture. The simpler of the synthetic antenna techniques is that which generates an unfocused synthetic aperture. In this case, the coherent signals received at the synthetic array points are integrated, with no attempt made to shift the phases of the signals before integration. This lack of phase adjustment imposes a maximum upon the synthetic antenna length that can be generated. This maximum synthetic antenna length occurs at a given range when the round-trip distance from a radar target to the center of the synthetic array differs by $\lambda/4$ from the round-trip distance between the radar target and the extremities of the synthetic aperture array.

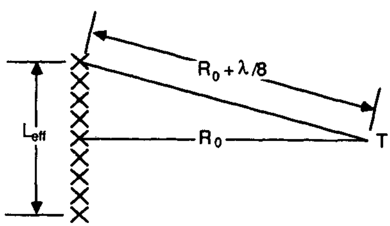

FIG. 21.2 Geometry for an unfocused synthetic antenna.

The pertinent geometry is shown in Fig. 21.2. In this figure, R_0 represents the range from a radar target to the center of the array, and L_{eff} represents the maximum synthetic antenna length such that the distance from the target to the extremities of the synthetic aperture does not exceed $R_0 + \lambda/8$.

It is evident from this geometry that

$$\left(R_0 + \frac{\lambda}{8}\right)^2 = \frac{L_{eff}^2}{4} + R_0^2 \tag{21.11}$$

If this expression is solved for L_{eff}, subject to the assumption that $\lambda/16$ is small compared with R_0, the result is

$$L_{eff} = \sqrt{R_0 \lambda} \tag{21.12}$$

Combination of Eqs. (21.2) and (21.12) gives

$$\beta_{eff} = \frac{1}{2}\sqrt{\frac{\lambda}{R_0}} \quad \text{rad} \tag{21.13}$$

Multiplying this beamwidth by range results in the resolution given by Eq. (21.7).

It will be noted that for the unfocused case the transverse linear resolution is independent of the antenna aperture size, fineness of resolution is increased by the use of shorter wavelengths, resolution varies as the square root of λ, and the resolution deteriorates as the square root of range. A plot of Eq. (21.7) is given in Fig. 21.1.

The Focused Case. An expression for the resolution achievable in the focused case has been given as Eq. (21.8). It is significant that the azimuth resolution achievable for this case depends only upon the physical antenna aperture and that, in contradistinction to the conventional case, fine resolution requires the use of small rather than large antennas. Also significant is the fact that the achievable resolution for a given antenna size is independent both of the range and of the wavelength used. A graph of Eq. (21.8) is also shown in Fig. 21.1.

In order to achieve the resolution indicated by Eq. (21.8), the synthetic aperture length required is

$$L_{eff} = \frac{\lambda R}{D} \tag{21.14}$$

The considerations used in arriving at Eq. (21.12) indicated that, unless additional processing were applied to the signals, antenna lengths such as those implied by Eq. (21.14) could not be achieved. The processing required is an adjustment of the phases of the signals received at each point of the synthetic antenna, which makes these signals of equal phase (cophase) for a given target. If this is done, the restrictions which limited the maximum antenna length to that given by Eq. (21.12) are no longer pertinent and the new limitation on the length of the synthetic antenna achievable becomes simply the linear width of the radiated beam at the range of the target.

In some cases, a resolution coarser than $D/2$ is sufficient. Then a fraction γ of the maximum focused synthetic antenna length can be used. For this case

$$L_{eff} = \frac{\gamma \lambda R}{D} \tag{21.15}$$

and the achievable resolution is

$$\text{Resolution}_{foc} = \frac{D}{2\gamma} \tag{21.16}$$

For situations in which the synthetic antenna length given by Eq. (21.15) is less than or equal to the synthetic antenna length for the unfocused case as given by Eq. (21.12), only a limited improvement in resolution is achievable for the focused case. However, if a resolution finer than that given by Eq. (21.7) is desired, focusing must be used. Focusing removes the restriction on synthetic aperture length that would otherwise apply.

21.3 RADAR SYSTEM PRELIMINARIES

Whether or not synthetic aperture generation is used, a number of components are required for a radar system. The use of a synthetic antenna and/or pulse compression places additional requirements on some of these components, especially with respect to coherence and stability.

It is the purpose of this section to present a block diagram of the portions of the radar system that precede the signal processor. A block diagram and several variants are described. The major variant is for the purpose of describing the transmitter-receiver portions of a radar system for the cases of synthetic antenna generation alone as compared with the case of synthetic antenna generation combined with pulse compression. The signal-processing operations will be discussed later.

The essential elements of a radar system useful in a synthetic aperture situation are shown in Fig. 21.3. The components that determine the radiated waveform are shown within the dotted lines in the upper left-hand corner of the diagram. This equipment consists of two stable oscillators. One of them is a local

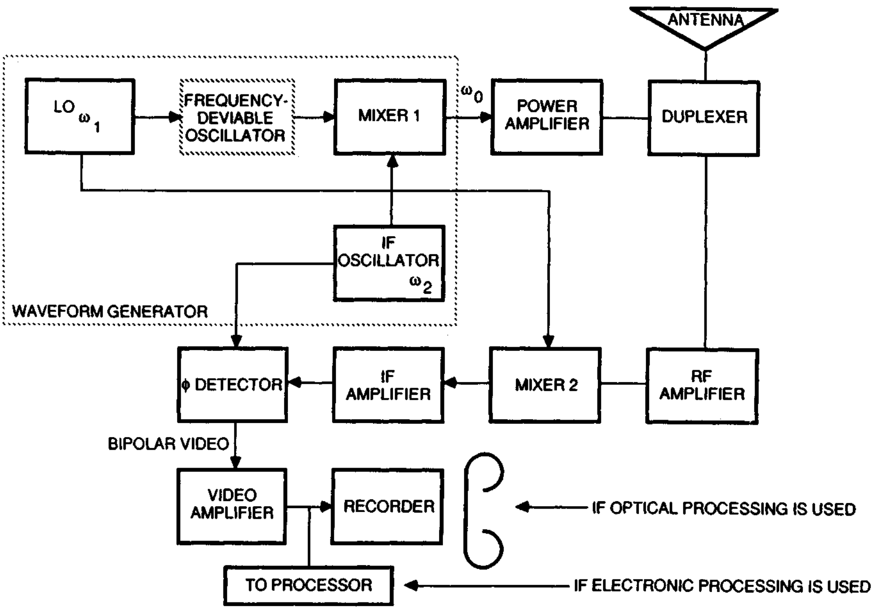

FIG. 21.3 Block diagram of a coherent radar system.

oscillator (LO) at radian frequency ω_2. The outputs of these oscillators are fed into mixer 1. In this mixer, a multiplicity of sum and difference frequencies is generated, and either the sum frequencies or the difference frequencies are selected and fed to the power amplifier.

If synthetic antenna generation without pulse compression is to be accomplished, the dotted component labeled "frequency-deviable oscillator" is not used, and the local oscillator is fed directly into mixer 1.

If pulse compression is to be combined with synthetic antenna generation, a frequency-deviable oscillator (FDO) is used for obtaining the desired waveform. In this case, the local oscillator is used to lock in the FDO. A ramp voltage is used to linearly frequency-modulate the FDO. This linearly frequency-modulated signal is then fed into mixer 1, instead of the LO signal, for the case of pulse compression. Waveforms other than linear frequency modulation may be used for pulse compression. In Fig. 21.3 the output of the video amplifier is fed to a recorder if optical processing is to be performed and/or to an electronic processor.

21.4 SIGNAL-PROCESSING THEORY

The theory of synthetic antenna generation combined with pulse compression is carried out below to show the information theoretic considerations involved and to indicate the operations necessary for achieving both azimuth synthetic antenna generation and pulse compression. A combined range-azimuth resolution function is derived. Following this treatment, an analysis of the signal-to-noise-ratio characteristics of a synthetic antenna compression radar are analyzed.

Detailed Resolution Analysis. The analysis will be carried out in terms of an ambiguity function whose properties indicate both the azimuth and the range resolution of the system. In the analysis, some conditions are stated for which the terms affecting range resolution can be factored from the elements affecting azimuth resolution, so that the resulting ambiguity function can be written as the product of two factors, one for range and one for azimuth.

Role of the Generalized Ambiguity Function. In this subsection, a definition of a generalized ambiguity function will be given, and its role in determining the resolution of a system will be interpreted.

To determine the generalized ambiguity function for a radar system, let a waveform $f(t)$ be radiated. We consider the operations performed upon the received signals with the objective of determining the radar reflectivity of the terrain being mapped. The function $f(t)$ may assume a variety of forms and may be a succession of short signals. If the quantity $\rho(x,y,z)$ represents the reflectivity of the terrain being mapped, the signal received by a radar system can be described by

$$s(t) = \int\int\int \rho(x,y,z)\, f\left[t - \frac{2R}{c}\right] dx\, dy\, dz \qquad (21.17)$$

The integration extends over the illuminated patch, and R is the range between a point (x,y,z) on the ground and the radar position $(vt,0,h)$. This equation shows that the received signal is the superposition of a large number of reflections within the illuminated pattern of the antenna and within the range gate which arrive simultaneously at the radar antenna.

The radar design problem is one of designing an operation on $s(t)$ to recover the reflectivity function $\rho(x,y,z)$. One such operation consists of passing the signals $s(t)$ through a matched filter. The operation of subjecting $s(t)$ to this matched filter is given by

$$e_0(R,R') = \int f^*\left[t - \frac{2R'}{c}\right] s(t)\, dt \tag{21.18}$$

In this equation, the asterisk indicates complex conjugation, and R' indicates the range from the radar antenna to the specific point (x',y',z') at which the reflectivity is to be evaluated.

Substitution of Eq. (21.17) into Eq. (21.18) gives a fourfold integral for the output, namely,

$$e_0 = \int\int\int\int \rho(x,y,z) f\left[t - \frac{2R}{c}\right] f^*\left[t - \frac{2R'}{c}\right] dt\, dx\, dy\, dz \tag{21.19}$$

If the order of integration can be inverted so that the integration with respect to t is performed first, one can define a quantity $\chi(x,y,z;x',y',z')$. This quantity is the generalized ambiguity function, given by

$$\chi(x,y,z;x',y',z') = \int f\left[t - \frac{2R}{c}\right] f^*\left[t - \frac{2R'}{c}\right] dt \tag{21.20}$$

In terms of the generalized ambiguity function defined by Eq. (21.17), Eq. (21.19) can be rewritten as

$$e_0 = \int\int\int \chi(x,y,z;x',y',z')\rho(x,y,z)\, dx\, dy\, dz \tag{21.21}$$

Equation (21.21) shows that the ambiguity function can be considered as a weighting function on $\rho(x,y,z)$. The output of the radar system, therefore, is the weighted average of ρ over a domain determined by the limits of integration. If the ambiguity function is localized at some point and is essentially zero at all other points, the output will be a good representation of the radar reflectivity at that point. Otherwise, the estimate of the reflectivity at a given point will be the weighted average given by Eq. (21.21).

Although no use will be made in this section of considerations determined by the limits of integration, it should be pointed out that Eq. (21.21) states that the output estimate of reflectivity in the radar system is a weighting of the reflectivity ρ by the product of the ambiguity function and the illumination function, whereby illumination function is meant the function that determines the distribution of signal energy over the plane. Ordinarily, the antenna illumination pattern, the pulse length, and the terms appearing in the radar equation determine this illumination function. In some cases, the ambiguity function χ has peaks at more than one point. If the illumination function excludes all but one of these peaks, an unambiguous system results.

Factorization of the Ambiguity Function. Let it be assumed that $f(t)$ can be written

$$f(t) = g(t)e^{i\omega_0 t} \tag{21.22}$$

In this equation, $g(t)$ is considered a complex function having both magnitude and phase, whereas ω_0 represents a carrier frequency. If $f(t)$ with the form given by Eq. (21.22) is used in Eq. (21.20), one obtains for the ambiguity function the expression

$$\chi = \int g\left[t - \frac{2R}{c}\right] g^*\left[t - \frac{2R'}{c}\right] e^{-i\omega_0(2R/c - 2R'/c)}\, dt \tag{21.23}$$

Let $f(t)$ consist of a sequence of transmissions. It is assumed that successive transmissions may be alike or that they may be different. Thus $f(t)$ will have the characteristics of being nonzero for a sequence of time intervals and of being zero otherwise. Further, let it be assumed that the exponential term in Eq. (21.23) varies slowly during each of these transmissions. This is equivalent to a statement that the electrical path length between a target and the radar changes by a small amount during each transmission. If this assumption is valid, the exponential term in Eq. (21.23) can be considered a constant during a given transmission, although it will vary between transmissions.

The integral that is the coefficient of the exponential term in Eq. (21.23) has the form of an autocorrelation function of g with itself. This autocorrelation function for g is given by Eq. (21.24). It will be noted that the autocorrelation function of g is a function of the difference in range $R - R'$. In Eq. (21.24) the integral is carried out over the times that $g(t - 2R/c)$ overlaps $g^*(t - 2R'/c)$ for a given transmission.

$$\phi_{gg} = \int g\left[t - \frac{2R}{c}\right] g^*\left[t - \frac{2R'}{c}\right] dt = \phi_{gg}\left[\frac{2R}{c} - \frac{2R'}{c}\right] \tag{21.24}$$

If the notation given by Eq. (21.24) is used, one obtains

$$\chi = \Sigma\phi_{gg}\left[\frac{2R}{c} - \frac{2R'}{c}\right] e^{-i\omega_0(2R/c - 2R'/c)} \tag{21.25}$$

Examination of Eq. (21.25) shows that if ϕ_{gg}, the autocorrelation function for g, is the same function for each member of the sequence of transmissions, then this element can be factored out and written outside the summation term of Eq. (21.25). The expression after this common term has been factored out is

$$\chi = \phi_{gg}\left[\frac{2R}{c} - \frac{2R'}{c}\right] \Sigma e^{-i\omega_0(2R/c - 2R'/c)} \tag{21.26}$$

The summation term in Eq. (21.26) gives the azimuth resolution of the system, whereas the term ϕ_{gg} gives the range resolution. It is evident from Eq. (21.26) that the autocorrelation function of g rather than g itself determines the range resolution of the system.

A variety of waveforms have been used to achieve range resolution. Among them the two most important are those in which $g(t)$ is a short pulse and those in which $g(t)$ is a linearly frequency-modulated short pulse (chirped signal). It is, of course, evident that any other waveform having a desirable autocorrelation function is equally possible for $g(t)$.

The form of $g(t)$ to be analyzed is that of the linearly frequency-modulated case.

Azimuth Resolution Factor of the Ambiguity Function. The azimuth resolution capability of the system can be determined from an evaluation of the sum term written as the factor in Eq. (21.26). In this sum, R represents the range from the radar to an arbitrary point on the terrain being mapped, whereas R' represents the range to a specific point for which the reflectivity is to be estimated. The geometry appropriate to the synthetic antenna case is shown in Fig. 21.4. In this diagram, it is assumed that the aircraft carries a side-looking antenna and flies at height h and with velocity v along the x axis so that the aircraft location is given by

$$x = vt$$

Consider two points being mapped having coordinates $(0,y_0,0)$ and $(x',y_0,0)$. R_0 is defined by

$$R_0 = \sqrt{{y_0}^2 + h^2} \tag{21.27}$$

and R and R' can be written, respectively, as

$$R = \sqrt{{R_0}^2 + x^2} \approx R_0 + \frac{x^2}{2R_0} \tag{21.28}$$

$$R' = \sqrt{{R_0}^2 + (x - x')^2} \approx R_0 + \frac{(x - x')^2}{2R_0} \tag{21.29}$$

The second expression for Eqs. (21.28) and (21.29) is an approximation that is valid whenever the inequalities $x \ll R_0$, $(x - x') \ll R_0$ are satisfied. If the approximate forms for $R - R'$ as given by the approximate expressions in Eqs.

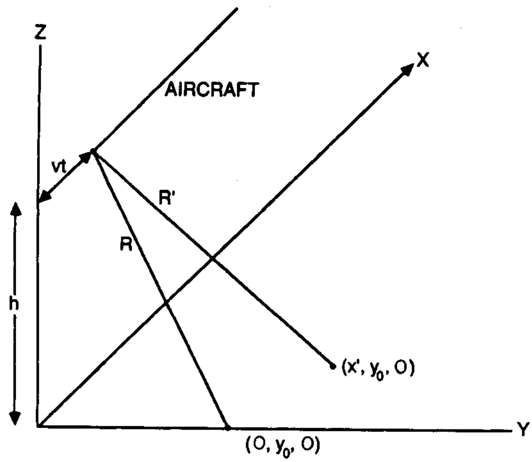

FIG. 21.4 Geometry for Eqs. (21.27) and (21.28).

(21.28) and (21.29) are used to evaluate the sum appearing in Eq. (21.26), one obtains

$$\Sigma e^{-i\omega_0(2R/c\ -\ 2R'/c)} = \Sigma e^{-i(2\omega_0/c)(2xx'\ -\ x'^2)/2R_0} \tag{21.30}$$

Thus far, the summation index has not been defined. To proceed further, it is necessary to indicate the summation index and its bounds. Let it be assumed that the transmissions radiated are a sequence of pulses with time intervals between pulses that are multiples of T. Then the variable x can be given as an integral multiple of the distance vT moved between successive transmissions. This relationship is

$$x = nvT \tag{21.31}$$

If Eq. (21.31) is substituted into Eq. (21.30), one obtains

$$\Sigma = e^{i(2\omega_0/c)x'^2/2R_0} \sum_{-N/2}^{N/2} e^{-i4\pi(x'/\lambda R_0)nvT} \tag{21.32}$$

In writing Eq. (21.32), the summation is carried over $N + 1$ terms. The synthetic antenna length implied by these limits is given by $L = NvT$.

Inasmuch as the summation terms in Eq. (21.32) are those corresponding to a geometric progression, the sum term can be immediately evaluated. The result is

$$\Sigma = e^{i(2\omega_0/c)x'^2/2R_0} \frac{\sin\,[(N + 1)4\pi x'vT/2\lambda R_0]}{\sin\,[4\pi x'vT/2\lambda R_0]} \tag{21.33}$$

The right-hand side of Eq. (21.33) gives the factor of the generalized ambiguity function that is responsible for the azimuth resolution. It will be noted that there are a phase term given by the exponential and a magnitude term given by the remaining terms in Eq. (21.33). The specific form of Eq. (21.33) is a consequence of the equal weighting of the signals. A weighting function can be used to shape the sidelobes in direct analogy with the use of such a technique in real antenna design.

Range Resolution Factor of the Ambiguity Function. This subsection considers the factor in the generalized ambiguity function, Eq. (21.25), that is responsible for range resolution. This factor, ϕ_{gg}, has been defined by Eq. (21.24). A specific form for $g(t)$ will be assumed, and an evaluation of ϕ_{gg} will be made for this specific waveform. The function $g(t)$ to be analyzed is that in which each radiation consists of a short, linearly frequency-modulated signal. An expression for $g(t)$ in this case is

$$g(t) = e^{i\alpha t^2} \tag{21.34}$$

The use of Eq. (21.34) in Eq. (21.24) results in

$$\phi_{gg} = e^{i\alpha[(2R/c)^2\ -\ (2R'/c)^2]} \int_{-\tau/2}^{\tau/2} e^{-i\alpha[(4R/c)t-(4R'/c)t]}\,dt \tag{21.35}$$

$$\phi_{gg} = e^{i\alpha[(2R/c)^2 - (2R'/c)^2]} \frac{\tau \sin \{\alpha\tau[(2R/c) - (2R'/c)]\}}{\alpha\tau[(2R/c - 2R'/c)]} \tag{21.36}$$

Equation (21.36) gives the range resolution factor of the ambiguity function for a transmitted waveform of the type expressed by Eq. (21.34). It will be noted that this term consists of a phase term and an amplitude term.

Inasmuch as both the azimuth resolution factor and the range resolution factor have been evaluated, the generalized ambiguity function can be written:

$$\chi = e^{i\alpha[(2R/c)^2 - (2R'/c)^2]} \frac{\tau \sin \alpha\tau[(2R/c) - (2R'/c)]}{\alpha\tau(2R/c - 2R'/c)} e^{i(2\omega_0/c)(x'^2/2R_0)} \times \frac{\sin [(N + 1)(2\pi x'vT)/(\lambda R_0]}{\sin [2\pi x'vT/\lambda R_0]} \tag{21.37}$$

In interpreting Eq. (21.37), it should be noted that there are phase terms and magnitude terms. One of the magnitude terms corresponds to the range resolution capability of the system; the other, to azimuth resolution. Quantitative expressions for the resolution in each of these coordinates will be obtained below.

The resolution terms in Eqs. (21.33), (21.36), (21.37) are of the form

$$\sin Nz/\sin z \tag{21.38}$$

with

$$z = 2\pi x'vT/\lambda R_0 \tag{21.39}$$

for the azimuth resolution case, and

$$\sin z/z \tag{21.40}$$

with

$$z = \alpha\tau[2R/c - 2R'/c] \tag{21.41}$$

for the range resolution case.

If the value

$$L = (N + 1)VT \tag{21.42}$$

is combined with Eq. (21.39) and one recognizes that

$$\alpha\tau = 2\pi B \tag{21.43}$$

where B is the chirp signal bandwidth, one can show that the azimuth resolution δ_a and the range resolution δ_r are given by

$$\delta_a = (1.4\lambda R_0)/(\pi L) \tag{21.44}$$

and

$$\delta_r = (1.4c)/(2\pi B) \tag{21.45}$$

Ambiguities. It is the objective of this subsection to make some observations regarding the possibility of multiple peaks (ambiguities) in the ambiguity function as given by Eq. (21.37) and its effect on system performance as given by Eq. (21.19).

The final term on the right-hand side of Eq. (21.37) is of the form

$$\frac{\sin [(N + 1)q]}{\sin q} \tag{21.46}$$

where the quantity q is defined by

$$q = \frac{2\pi x' vT}{\lambda R_0} \tag{21.47}$$

The azimuth resolution factor, therefore, has a peak whenever the quantity q takes on a value equal to an integral multiple of π rad. Thus the system is potentially ambiguous for values of x' that are solutions of

$$\frac{2\pi x' vT}{\lambda R_0} = m\pi \tag{21.48}$$

Actually, it is more meaningful to solve for the ratio x'/R_0. The angle υ gives directions from the broadside at which angle ambiguities are potentially possible. This relationship is given as

$$\sin \theta = \frac{x'}{R_0} = \frac{\lambda}{D} \frac{D}{vT} \frac{m}{2} = \frac{m\beta D}{2vT} \tag{21.49}$$

In the last form of this equation, the numerator and the denominator have been multiplied by D, the horizontal aperture of the antenna, to express the result in terms of the radiated beamwidth $\beta = \lambda/D$.

Thus, the possibility of azimuth ambiguities arises as a natural consequence of the signals radiated and of the processing method. Ordinarily, these potential ambiguities in azimuth are suppressed by the illumination factor. The illumination pattern β is chosen so that the values of β corresponding to more than one value of m are not illuminated.

Possibilities also exist for ambiguities in range. The analysis carried out to the point of Eq. (21.37) was not sufficiently general to predict ambiguities in range. However, if reference is made to Eq. (21.24), it is evident that the autocorrelation function ϕ_{gg} will be periodic if $g(t)$ is periodic. Thus range ambiguities can also occur. In particular, range ambiguities will occur for ranges having a difference given by

$$\Delta R = \frac{cT}{2} \tag{21.50}$$

where T is the interpulse period.

To date, systems have been built which have avoided ambiguities by virtue of illuminating only the part of the ambiguity diagram that excludes all but one major peak. This technique has sometimes been referred to as *ambiguity avoidance.*

For some sets of parameters, ambiguities cannot be avoided by using a radar with a single radiated beam. The use of multiple beams solves this problem. This topic is discussed in Sec. 21.5.

Signal-to-Noise-Ratio Considerations. It is the purpose of this subsection to derive expressions for signal-to-noise (S/N) ratio for radars in which pulse compression and synthetic antenna techniques are used. The signal-to-noise ratio for a radar system as a result of the reception of a single pulse is given by the well-known radar equation

$$\frac{S}{N} = \frac{P_t G_t A_r \sigma}{(4\pi)^2 R^4 k T_0 B F_n} \tag{21.51}$$

In a pulse compression radar, signal-to-noise improvement occurs in the ratio of the uncompressed pulse length τ_i to compressed pulse length τ_0.

In a radar that achieves its azimuth resolution by the generation of a synthetic antenna, there is an additional signal-to-noise improvement factor due to the integration of a number of pulses. The number of pulses integrated is equal to the product of the pulse repetition frequency (PRF) and the time necessary to generate the synthetic antenna. In turn, this time is equal to the ratio of synthetic length L to aircraft speed v.

An expression in which the product of both factors has been written is

$$\text{Improvement factor} = \frac{\tau_i}{\tau_0} \frac{\text{PRF } L}{v} \tag{21.52}$$

The length of synthetic antenna required to achieve azimuth resolution δ_{az} at range R and wavelength λ is given by

$$L = \frac{R\lambda}{2\delta_{az}} \tag{21.53}$$

The substitution of Eq. (21.53) into Eq. (21.52) gives for the improvement factor

$$\text{Improvement factor} = \left[\frac{\tau_i}{\tau_0}\right] \frac{\text{PRF } R\lambda}{2v\delta_{az}} \tag{21.54}$$

The signal-to-noise ratio including the improvement factor is obtained by multiplying together the expressions given by Eqs. (21.51) and (21.54). The result of this multiplication is

$$\frac{S}{N} = \frac{P_t G_t A_r \sigma}{(4\pi)^2 R^4 k T_0 B F_n} \frac{\tau_i}{\tau_0} \frac{\text{PRF } R\lambda}{2v\delta_{az}} \tag{21.55}$$

Although Eq. (21.55) contains the desired information, it is useful to modify the term somewhat by expressing the antenna gain in terms of the effective area of its aperture and of the wavelength. This expression is written

$$G_t = \frac{4\pi A_r}{\lambda^2} \tag{21.56}$$

It is also desirable to collect together three terms in the numerator of Eq. (21.55), namely, P_t, the peak transmitted power; τ_i, the uncompressed pulse length; and the PRF. The product of these three factors gives the average power P_{av}. This relationship is written

$$P_{av} = P_t \tau_i \text{PRF} \tag{21.57}$$

In the design of a radar system, the bandwidth B is chosen to be the reciprocal of τ_0. Hence the product of the bandwidth and the compressed pulse width is approximately equal to unity. This relationship is written

$$B\tau_0 \approx 1 \tag{21.58}$$

Finally, it is useful to express the radar cross section σ in terms of the azimuth and range resolution, δ_{az} and δ_r, as well as in terms of the reflectivity of the terrain, ρ. The radar cross section is equal to the reflectivity of the terrain multiplied by the projected area. This projection accounts for the term $\sin \psi$. The expression for the radar cross section in terms of these parameters is given by

$$\sigma = \rho \delta_r \delta_{az} \sin \psi \tag{21.59}$$

Substitution of Eqs. (21.56) to (21.59) for the corresponding quantities in Eq. (21.55) gives

$$\frac{S}{N} = \frac{P_t \sigma_i \text{PRF} 4\pi A^2 \rho \delta_r \delta_{az} (\sin \psi) R\lambda}{(4\pi)^2 R^4 \lambda^2 k T_0 F_n (B\tau_0) 2v \delta_{az}} \tag{21.60}$$

In writing Eq. (21.60), no cancellation of terms has been made. Canceling terms that appear in both the numerator and the denominator results in

$$\frac{S}{N} = \frac{P_{av}}{8\pi} \frac{A_r^2 \rho \delta_r}{k T_0 F_n R^3 \lambda} \frac{\sin \psi}{v} \tag{21.61}$$

This is the desired result.

Equation (21.61) does not take into account factors concerned with ambiguity avoidance. The inclusion of such effects is given in Ref. 5.

Equation (21.61) shows that the signal-to-noise ratio at the output of a radar that has used pulse compression and has generated a synthetic antenna has the following properties different from conventional radar:

1. The signal-to-noise ratio is proportional to the size of the range resolution element and is independent of the size of the azimuth resolution element.

2. The signal-to-noise ratio is inversely proportional to the third power of range.

3. The signal-to-noise ratio is inversely proportional to the wavelength.

4. The signal-to-noise ratio is inversely proportional to the speed of the aircraft.

Effect of Phase Errors. In actual equipment, phase errors arise from a number of sources. Some of the instabilities arise in oscillators and other electrical components of the radar, but other sources of phase error are inhomogeneities in the atmosphere or the result of uncompensated deviation of the aircraft from linear unaccelerated motion. A number of modifications of the synthetic antenna pattern result from such uncompensated phase errors. These modifications include beam canting, beam spreading, peak gain reduction, and redistribution of the ratio of energy in the main lobe to that in the sidelobes. An analytic formulation and Monte Carlo computer simulation of the effects of phase errors for normally distributed random phase errors and for three cross-correlation functions have been given by Greene and Moller.[6]

Signal Processing. The preceding subsections have discussed a number of aspects of radar signal processing. Also discussed has been the radar system up to the point of signal processing. As part of that analysis, the waveforms of signals at a number of points in the radar system have been described. It is the purpose of this subsection to discuss a number of aspects of signal processing that are common to all mechanizations.

Many fine-resolution radar systems employ both pulse compression and synthetic antenna generation.

Theoretic Aspects of Synthetic Aperture Generation. In generating a synthetic antenna, the returns from a number of spatial positions must be combined. In doing this, one usually wishes to apply weighting to the signals for synthetic antenna pattern sidelobe-level control; in the case of focused synthetic antennas, one also wishes to adjust the phases of the signals before combination.

In the preceding discussion, the signal was represented as a function of time. For present purposes it is preferable to consider the signals as a discrete sequence numbered from 1 to N. Let S_n represent the signal received when the physical antenna is at the nth position of the antenna array. Let W_n represent weighting applied to S_n, and let ϕ_n represent the phase adjustment required for focusing.

The operation of synthetic antenna generation then consists of taking the vector sum of the signals S_n, adjusting their phases, and multiplying by weighting factors. The sum of this operation is given by

$$\Sigma S_n e^{i\phi_n} W_n = \text{focused pattern} \tag{21.62}$$

In the case of unfocused synthetic antenna generation the phase adjustments ϕ_n are not made. In this case the signal operations required have the form

$$\Sigma S_n W_n = \text{unfocused synthetic pattern} \tag{21.63}$$

There are many mechanizations possible to carry out the operations indicated by Eqs. (21.62) and (21.63). Some of them are described below. Two common techniques are digital and optical in nature.

Discussions regarding optical and digital data processing are given in Refs. 7 through 11.

Optical Techniques. The optical techniques involve the recording of the radar signals on a transparency, most frequently silver halide photographic film in any of a number of formats. Initially the successive range sweeps were placed parallel and side by side; later polar format was used. The growing understanding that we are really collecting a portion of the three-dimensional spectrum has led to use of a three-dimensional storage format.[8] This topic will be discussed further in Sec. 21.5.

The most frequently used optical processors are based on the tilted-plane optical processor described by Kozma, Leith, and Massey.[7] In this processor, both the input plane and the output plane are tilted (i.e., they are not perpendicular to the optical axis). The optical components are telescopic, and the powers of the elements in two perpendicular planes are unequal. The telescopic elements include both spherical and cylindrical elements.

The evolution of this processor is based in part upon the recognition that signal histories may be assigned focal lengths and behave to some degree as optical elements.

The azimuth along-track signals from a point target are similar to those of a zone plate. The focal length is proportional to target range. If pulse compression is used, all targets have associated with them zone plates in the range direction. These all have the same focal length. The recorded signals before processing are often referred to as *signal histories.*

Digital Processors. Digital processing has emerged as the preferred means when the amount of data to be processed is not too great. Mechanization of SAR operations is often computation intensive.

In cases not based on polar format, correlation operations have been used. These operations are usually performed using frequency-plane equivalent of correlation.

$$\int f(q)\, g(q - x)\, dx = \int F(\omega)\, G(\omega) \exp (j\omega x)\, d\omega \qquad (21.64)$$

In other cases, such as polar format, Fourier transform operations are indicated and the use of the fast Fourier transform (FFT) plays an important role.

Associated with FFT processing is the fact that algorithms exist for processing two-dimensional data with sampling points in the rectangular arrays, whereas the sample points obtained are equally spaced on radial lines. This requires formatting operations on the data points to convert them to rectangular format.[12,13]

Imagery from synthetic aperture radars is shown in Figs. 21.5 and 21.6. These images were provided by the Environmental Research Institute of Michigan (ERIM).

21.5 ADDITIONAL SYSTEM CONSIDERATIONS

In this section a number of considerations peculiar to synthetic aperture radar are discussed. Some are additional performance requirements on the components of the system; some are concerned with system aspects.

Antenna. The horizontal aperture of the antenna determines the finest azimuth resolution achievable in a single-beam synthetic aperture radar except for the searchlight mode. Moreover, in the signal processing it is assumed that the antenna gain is constant as a function of along-track position. Thus, it is necessary to have a degree of stabilization of antenna pointing so that the beam

FIG. 21.5 STAR-1 radar imagery, lower Lake St. Clair, upper Detroit River. Resolution, 20 ft (6 m). (*Courtesy Environmental Research Institute of Michigan.*)

rotation is some minor fraction of the beamwidth. In most cases, the antenna is side-looking, although in some cases the antenna is positioned to an angle off broadside and the system then operates in what is called the *squint* mode.

Receiver-Transmitter. The transmitter and receiver for synthetic antenna radars require maintenance of coherence of the radar signals. Consequently, there is emphasis on the stability of the oscillators and more rigorous requirements on the components. The output of coherent radar is the output of a synchronous demodulator rather than that of an envelope detector commonly used in radars. The output is bipolar video, in which a reference bias level corresponds to zero level of video output.

Storage and Recording. It is inherent that synthetic antenna radars and pulse compression radars require the storage of radar data, because the *data* for synthetic antenna generation does not occur simultaneously but is collected over some interval of time. Operations are then performed on these signals to achieve the selectivity of the radar. Moreover, each radar return participates in forming the output for a large number of points on the output map. The requirements for storage are therefore very large. Since a high volume of storage is required for a fine-resolution radar system, photographic storage is commonly used.

For digital processing, storage of the digital signals after analog-to-digital (A/D) conversion is required. The amount of this data can be great and often limits the area over which fine resolution can be obtained. A description of what needs to be done is given in Refs. 12 and 13.

In selecting a storage medium one must consider the rate at which information must be recorded, the amount of data to be recorded, and the rate at which the

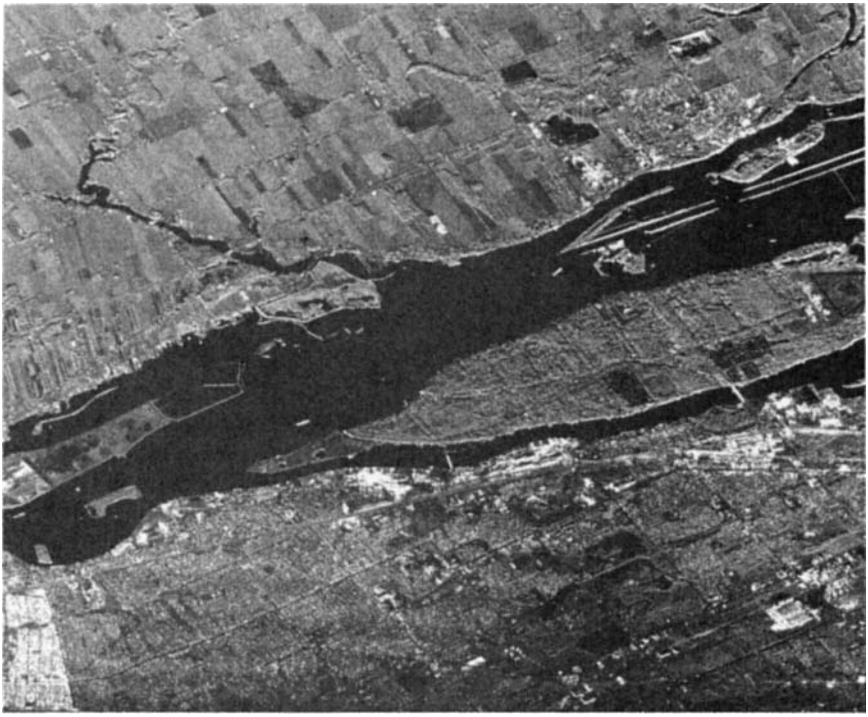

FIG. 21.6 STAR-1 radar imagery, lower Detroit River. Resolution, 20 ft (6 m). (*Courtesy of Environmental Research Institute of Michigan.*)

storage must be read out for performing the azimuth compression and the pulse compression.

Motion Compensation. In generating the synthetic antenna the signal-processing equipment assumes that the radar flies along a straight line at constant speed. In practice, the vehicle carrying the antenna is subject to deviations from unaccelerated flight. Therefore, it is necessary to have auxiliary equipment to compensate for other than straight-line motion. Motion-compensation equipment must include sensors capable of detecting the deviation of the flight path from a linear path. The output of these sensors is used in a variety of ways. For motion compensation proper, the received-signal phase must be adjusted to compensate for the displacement of the real antenna from the location of the ideal synthetic antenna being generated.

A consideration of the geometry involved shows that the phase correction that must be applied is a function of depression angle. Consequently, the correction must be made as a function of range. The rate of change correction is very rapid at steep depression angles and becomes slower at shallow depression angles.

Squint Mode. In most examples of synthetic aperture radar, the beam is directed at right angles to the ground track of the aircraft. In some cases, however, it is desirable to "squint" the antenna beam so that an area either forward or aft of the aircraft is mapped. It is necessary to position the antenna beam so that the maximum of its radiation pattern points in the desired squint direction. Moreover, it is usually necessary to modify the signal processors to take into account the average doppler frequency shift that occurs when the antenna points in a direction other than normal to the flight path. It is, of course, also necessary to take the geometry of the squint mode into account in designing recorders and displays.

Spotlight Mode. In Sec. 21.2 Eq. (21.8) was derived for a radar in *strip-map* mode, i.e., for the case that the radar antenna is in a fixed orientation and the radar beamwidth $R\lambda/D$ is used as the length of synthetic aperture generated. One can increase the antenna length by use of *spotlight* mode. In this case the radar antenna is continuously pointed toward the region being imaged. For this case one can make a synthetic antenna length longer than $R\lambda/D$, or one can make several images and noncoherently integrate them.

Spotlight mode also makes possible the use of higher antenna gain.

Effects of Motion Errors. In generating synthetic aperture antenna images, one needs to estimate the along-track and cross-track velocities of targets in order to derive the matched filter to use in imaging. If one has an error in the radial velocity of the target, one gets a rotation in target position. If one has an error in along-track velocity, a limit is set to the achievable resolution.

Multiple-Beam Radars. The analyses leading to Eqs. (21.5) and (21.8) are correct for synthetic aperture radars which radiate only one beam. However, system parameters sometimes dictate the use of multiple beams.

The use of multiple beams is motivated by several considerations, such as ambiguity avoidance and the achievement of higher antenna gain. The achievement of a larger area coverage rate is another but less likely motivation. A great deal of flexibility is possible. The multiple beams may be arranged in the azimuth direction or the range direction, or both. With the use of multiple beams, it is possible to achieve any desired combination of unambiguous range, resolution, and area rate. The antenna area and the number of beams are determined from the performance parameters.

A more complete analysis of multibeam systems is given in Refs. 14 through 17.

ISAR. *Inverse synthetic aperture radar* (ISAR) is the term used when the motion of the object being imaged is used instead of the motion of the radar. A more general case is that in which both the radar and the object are in motion. In ISAR, the target motion is often not known to the radar. Hence, a major part of the problem is determination of the target motion to generate the matched filter needed to generate an image. A number of techniques have been studied for providing data regarding both translation and rotation of moving objects. An example of such work is that of B. Steinberg.[18]

Three-Dimensional Spectrum. The analysis starting with Eqs. (21.15) and (21.16) contains the assumption that a matched filter is applied to the radar returns from each point. This can in fact be done. It would reduce the signal-processing load if a reference function could be applied over a region. This, too, can be done, but range walk and defocusing set limits to the size of a patch which can be handled in this manner. Of the methods that have been proposed, the use of polar format and its generalization and the collection of a portion of the three-dimensional spectrum of the scene being mapped are most significant.

If one starts with an expression such as Eq. (21.17), performs the integration with respect to τ, where τ is a dummy variable to replace t, and then makes a Fourier transform of the results, one gets

$$E_0(\omega) = \int \rho(x,y,z)|G(\omega)|^2 \exp\left[-j(2\omega/c)(2r/c-2r'/c)\right] dx\, dy\, dz \tag{21.65}$$

In this equation $|G(\omega)|^2$ is the Fourier transform of the autocorrelation function of $g(t)$. Let the vector difference, or r, and r' be represented by q.

$$\mathbf{r} = \mathbf{r}' + \mathbf{q} \tag{21.66}$$

so that

$$r = r' + \frac{r' \cdot q}{r'} + \frac{q^2 + (r' \cdot q/r')^2}{2r'} \tag{21.67}$$

If one can neglect the last term, and if one writes

$$\hat{\mathbf{r}} = \mathbf{r}/\mathbf{r} \tag{21.68}$$

for the unit vector along r, one can write

$$r - r' = \hat{\mathbf{r}} \,.\, \mathbf{q} \tag{21.69}$$

Let a vector G be defined by

$$\mathrm{G} = (2\ \omega/c)\hat{\mathbf{r}} \tag{21.70}$$

then

$$E_0(\omega) = \int \rho(x,y,z) G(\omega)^2 \exp\left(-jG \cdot \mathbf{q}\right) dq \tag{21.71}$$

One notes that except for the factor $|G(\omega)|^2$, Eq. (21.71) has the form of a three-dimensional spectrum of $\rho(\mathbf{q})$.

Equation (21.71) has been derived by Jack Walker.[8] Related developments involving use of the *projection-slice theorem* have been given by a number of other authors.[9,19]

In interpreting Eq. (21.71), it is useful to consider the point **r** as a general point on the object being imaged and **r**′ as a reference point on the object. The vector **q** is then the vector from the reference point to all other points on the object, and the integration extends over the object.

Equation (21.71), being a three-dimensional spectrum of an object, requires that the data be taken with an origin of coordinates and a coordinate system fixed with respect to the object being imaged. Hence the effect of translation and rotation of a moving object must be compensated in order to image that object.

The three-dimensional spectrum is the proper format for storing radar data. The polar format is a special case in which the radar collection is performed in a plane. Use of the projection-slice theorem enables one to project the data along any direction. This promises to give images of *slices* of moving objects.

The projection of the radar data, followed by a two-dimensional Fourier transform, can be used to form images for the most general motion of both the radar and the moving object.

REFERENCES

1. Sherwin, C. W., P. Ruina, and R. D. Rawcliffe: Some Early Developments in Synthetic Aperture Radar Systems, *IRE Trans.*, vol. MIL-6, pp. 111–115, April 1962.
2. Wiley, C.: Pioneer Award acceptance remarks, *IEEE Trans.*, vol. AES-21, pp. 433–443, May 1986.
3. Cutrona, L. J., W. E. Vivian, E. N. Leith, and G. O. Hall: A High Resolution Radar Combat-Surveillance System, *IRE Trans.*, vol. MIL-5, pp. 127–131, April 1961.
4. Cutrona, L. J., and G. O. Hall: A Comparison of Techniques for Achieving Fine Azimuth Resolution, *IRE Trans.*, vol. MIL-6, pp. 119–121, April 1962.
5. Skolnik, M. I.: "Introduction to Radar Systems," 2d ed., McGraw-Hill Book Company, 1980, p. 522, Eqs. 14 and 16.
6. Greene, C. A., and R. T. Moller: The Effect of Normally Distributed, Random Phase Errors on Synthetic Array Gain Patterns, *IRE Trans.*, vol. MIL-6, pp. 130–139, April 1962.
7. Kozma, A., E. N. Leith, and N. G. Massey: Tilted Plane Optical Processor, *Appl. Opt.*, vol. 11, pp. 1766–1777, August 1972.
8. Walker, J. L.: Range Doppler Imaging of Rotating Objects, *IEEE Trans.*, vol. AES-16, pp. 23–52, January 1980.
9. Ausherman, D. A., A. Kozma, J. L. Walker, H. M. Jones, and E. C. Poggio: Developments in Radar Imaging, *IEEE Trans.*, vol. AES-20, pp. 363–400, July 1984.
10. Cutrona, L. J., E. N. Leith, C. J. Palermo, and L. J. Porcello: Optical Data Processing and Filtering System, *IRE Trans.*, vol. IT-6, pp. 386–400, June 1960.
11. McLeod, J.: The Axicon: A New Type of Optical Element, *J. Opt. Soc. Am.*, vol. 44, pp. 592–597, August 1954.
12. Ausherman, D. A.: Digital Image Processing, *Proc. SPIE*, vol. 528, pp. 118–133, Jan. 22–23, 1985.
13. Ausherman, D. A.: Digital versus Optical Techniques in Synthetic Aperture Radar (SAR) Data Processing, *Opt. Eng.*, vol. 19, pp. 157–167, March–April 1980.
14. Cutrona, L. J.: Means to Achieve Wide Swath Widths in Synthetic Aperture Radar, *Addendum to Proc. Synth. Aperture Radar Technol. Conf.*, pp. V-9-1–V-9-21, New Mexico State University, Las Cruces, Mar. 8–10, 1978.
15. Claassen, J. P., and J. Eckerman: A System Concept for Wide Swath Constant Incident Angle Coverage, *Proc. Synth. Aperture Radar Technol. Conf.*, Pap. VI-4, pp. VI-4-1–VI-4-19, New Mexico State University, Las Cruces, Mar. 8–10, 1978.
16. Cutrona, L. J.: Comparison of Sonar System Performance Achievable Using Synthetic-Aperture Techniques with the Performance Achievable by More Conventional Means, *J. Acoust. Soc. Am.*, vol. 58, pp. 336–348, August 1975.
17. Cutrona, L. J.: Additional Characteristics of Synthetic Aperture Sonar Systems and a Further Comparison with Non-Synthetic Aperture Sonar Systems, *J. Acoust. Soc. Am.*, vol. 61, pp. 1213–1217, May 1977.
18. Steinberg, B. D.: "Microwave Imaging with Large Antenna Arrays," John Wiley & Sons, New York, 1983.
19. Munson, D. C., Jr., J. D. O'Brien, and W. K. Jenkins: A Topographic Formulation of Spotlight-Mode Synthetic Aperture Radar, *Proc. IEEE*, vol. 71, pp. 917–925, August 1983.